INTEGRAL REPRESENTATIONS AND ASYMPTOTIC BEHAVIOUR OF A MITTAG–LEFFLER TYPE FUNCTION OF TWO VARIABLES

CHRISTIAN LAVAULT

Communicated by C. Lizama

Abstract. Integral representations play a prominent role in the analysis of entire functions. The representations of generalized Mittag–Leffler type functions and their asymptotics have been (and still are) investigated by plenty of authors in various conditions and cases.

The present paper explores the integral representations of a special function extending to two variables the two-parametric Mittag–Leffler type function. Integral representations of this functions within different variation ranges of its arguments for certain values of the parameters are thus obtained. Asymptotic expansion formulas and asymptotic properties of this function are also established for large values of the variables. This yields corresponding theorems providing integral representations as well as expansion formulas.

References

1. M. M. Džrbašjan, *On the integral transformations generated by generalized functions of the Mittag–Leffler type* (in Russian), Izv. Akad. Nauk Armjan. SSR Ser. Fiz.-Mat. Nauk **13** (1960), no. 3, 21–63.
2. M. M. Džrbašjan, *Integral transforms and representations of functions in the complex domain* (in Russian), Izdat. "Nauka", Moscow 1966, 671 pp.
3. A. Erdélyi, W. Magnus, F. Oberhettinger, and F. G. Tricomi, *Higher Transcendental Functions*, Based, in part, on notes left by Harry Bateman, McGraw-Hill Book Company, Inc., New York-Toronto-London, 1955.

4. M. Garg, P. Manohar, and S. L. Kalla, *A Mittag–Leffler function of two variables*, Integral Transforms Spec. Funct. **24** (2013), no. 11, 934–944.

5. R. Gorenflo, A. A. Kilbas, F. Mainardi, and S. V. Rogosin, *Mittag–Leffler functions, related topics and applications*,Springer Monographs in Mathematics, Springer, Heidelberg, 2014.

6. R. Gorenflo, J. Loutchko, and Yu. Luchko, *Computation of the Mittag–Leffler function $E_{\alpha,\beta}(z)$ and its derivatives*, Fract. Calc. Appl. Anal. **5** (2002), no. 4, 491–233.

7. H. Haubold, A. M. Mathai, and R. K. Saxena, *Mittag–Leffler functions and their applications*, J. Appl. Math. **2011**, Art. ID 298628, 51 pp.

8. P. Humbert and R. P. Agarwal, *Sur la fonction de Mittag–Leffler et quelques-unes de ses généralisations*, Bull. Sci. Math. **77** (1953), no. 2, 180–185.

9. C. Lavault, *Fractional calculus and generalized Mittag–Leffler type functions*, preprint (2017), arXiv:1703.01912 & HAL:hal-01482060.

10. E. N. Ogorodnikov and N. S. Yashagin, *Setting and solving of the Cauchy type problems for the second order differential equations with Riemann–Liouville fractional derivatives*, Vestn. Samar. Gos. Tekhn. Univ. Ser. Fiz.-Mat. Nauki **1** (2010), no. 20, 24–36.

11. N. M. Temme, *Special functions. An introduction to the classical functions of mathematical physics*, A Wiley-Interscience Publication, John Wiley & Sons, Inc., New York, 1996.

12. A. Wiman, *Über den fundamentalsatz in der theorie der funktionen $E_a(z)$*, Acta Math. **29** (1905), no. 1, 191–201.

LIPN, CNRS UMR 7030, Université Paris 13, Sorbonne Paris Cité, F-93430 Villetaneuse, France.

E-mail address: lavault@lipn.univ-paris13.fr