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ON LINEAR MAPS PRESERVING CERTAIN PSEUDOSPECTRUM AND CONDITION SPECTRUM SUBSETS

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Abstract. We define two new types of spectrum, called the ε-left (or right) pseudospectrum and the ε-left (or right) condition spectrum, of an element a in a complex unital Banach algebra A. We prove some basic properties among them the property that the ε-left (or right) condition spectrum is a particular case of Ransford spectrum. We study also the linear preserver problem for our defined functions and we establish the following:

(1) Let A and B be complex unital Banach algebras and $\varepsilon > 0$. Let $\phi : A \longrightarrow B$ be an ε-left (or right) pseudospectrum preserving onto linear map. Then ϕ preserves certain standard spectral functions.
(2) Let A and B be complex unital Banach algebras and $0 < \varepsilon < 1$. Let $\phi : A \longrightarrow B$ be a unital linear map. Then
(a) If ϕ is an ε-almost multiplicative map, then $\sigma^l(\phi(a)) \subseteq \sigma^l_\varepsilon(a)$ and $\sigma^r(\phi(a)) \subseteq \sigma^r_\varepsilon(a)$, for all $a \in A$.
(b)If ϕ is an ε-left (or right) condition spectrum preserving, then (i) if A is semi-simple, then ϕ is injective; (ii) if B is spectrally normed, then ϕ is continuous.

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