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CERTAIN GEOMETRIC STRUCTURES OF Λ-SEQUENCE SPACES

ATANU MANNA

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Abstract. The Λ-sequence spaces Λ_p for $1 < p \leq \infty$ and their generalized forms $\Lambda_{\hat{p}}$ for $1 < \hat{p} < \infty$, $\hat{p} = (p_n)$, $n \in \mathbb{N}_0$ are introduced. The James constants and strong n-th James constants of Λ_p for $1 < p \leq \infty$ are determined. It is proved that the generalized Λ-sequence space $\Lambda_{\hat{p}}$ is a closed subspace of the Nakano sequence space $l_{\hat{p}}(\mathbb{R}^{n+1})$ of finite dimensional Euclidean space $\mathbb{R}^{n+1}$, $n \in \mathbb{N}_0$. Hence it follows that sequence spaces Λ_p and $\Lambda_{\hat{p}}$ possess the uniform Opial property, (β)-property of Rolewicz, and weak uniform normal structure. Moreover, it is established that $\Lambda_{\hat{p}}$ possesses the coordinate wise uniform Kadec–Klee property. Further, necessary and sufficient conditions for element $x \in S(\Lambda_{\hat{p}})$ to be an extreme point of $B(\Lambda_{\hat{p}})$ are derived. Finally, estimation of von Neumann–Jordan and James constants of two dimensional Λ-sequence space $\Lambda_2^{(2)}$ are carried out. Upper bound for the Hausdorff matrix operator norm on the non-absolute type Λ-sequence spaces is also obtained.

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Faculty of Mathematics, Indian Institute of Carpet Technology, Chauri Road, Bhadohi- 221401, Uttar Pradesh, India.

E-mail address: atanu.manna@iict.ac.in or atanuiitkgp86@gmail.com