

PERTURBATION OF MINIMUM ATTAINING OPERATORS

JADAV GANESH, GOLLA RAMESH,* and DANIEL SUKUMAR

Communicated by R. Drnovšek

ABSTRACT. We prove that the minimum attaining property of a bounded linear operator on a Hilbert space H whose minimum modulus lies in the discrete spectrum, is stable under small compact perturbations. We also observe that given a bounded operator with strictly positive essential minimum modulus, the set of compact perturbations which fail to produce a minimum attaining operator is smaller than a nowhere dense set. In fact, it is a porous set in the ideal of all compact operators on H . Further, we try to extend these stability results to perturbations by all bounded linear operators with small norm and obtain subsequent results.

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Date: Received: Aug. 10, 2017; Accepted: Dec. 20, 2017.

*Corresponding author.

2010 *Mathematics Subject Classification.* Primary 47B07; Secondary 47A10, 47A55, 47B65.

Key words and phrases. Minimum modulus, spectrum, essential spectrum, porous set.

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DEPARTMENT OF MATHEMATICS, IIT HYDERABAD, KANDI, SANGAREDDY, TELANGANA 502285, INDIA.

E-mail address: ma12p1003@iith.ac.in

E-mail address: rameshg@iith.ac.in

E-mail address: suku@iith.ac.in