

WELL-POSEDNESS ISSUES FOR A CLASS OF COUPLED NONLINEAR SCHRÖDINGER EQUATIONS WITH CRITICAL EXPONENTIAL GROWTH

HANEN HEZZI

Communicated by J. D. Rossi

ABSTRACT. The initial value problem for some coupled nonlinear Schrödinger equations in two space dimensions with exponential growth is investigated. In the defocusing case, global well-posedness and scattering are obtained. In the focusing sign, global and nonglobal existence of solutions are discussed via potential well-method.

REFERENCES

1. S. Adachi and K. Tanaka, *Trudinger type inequalities in \mathbb{R}^N and their best exponent*, Proc. Amer. Math. Soc. **128** (2000), no. 7, 2051–2057.
2. H. Bahouri, S. Ibrahim, and G. Perelman, *Scattering for the critical 2-D NLS with exponential growth*, Differential Integral Equations **27** (2014), no. 3-4, 233–268.
3. T. Cazenave, *An introduction to nonlinear Schrödinger equations*, Textos de Metodos Matematicos 26, Instituto de Matematica UFRJ, 1996.
4. J. Colliander, M. Grillakis, and N. Tzirakis, *Tensor products and correlation estimates with applications to nonlinear Schrödinger equations*, Comm. Pure Appl. Math. **62** (2009), 920–968.
5. J. Colliander, S. Ibrahim, M. Majdoub, and N. Masmoudi, *Energy critical NLS in two space dimensions*, J. Hyperbolic Differ. Equ. **6** (2009), no. 3, 549–575.
6. J. Colliander, M. Keel, G. Staffilani, H. Takaoka, and T. Tao, *Global well-posedness and scattering for the energy-critical nonlinear Schrödinger equation in \mathbb{R}^3* , Ann. of Math. (2) **167** (2008), no. 3, 767–865.

Copyright 2018 by the Tusi Mathematical Research Group.

Date: Received: Sep. 7, 2017; Accepted: Feb. 7, 2018.

2010 *Mathematics Subject Classification*. Primary 35Q55; Secondary 35Q41.

Key words and phrases. Nonlinear Schrödinger system, global well-posedness, scattering, blow-up, Moser–Trudinger inequality.

7. J. F. Lam, B. Lippmann, and F. Tappert, *Self trapped laser beams in plasma*, Phys. Fluid. **20** (1997), no. 7, 1176–1179.
8. L. Ma and L. Zhao, *Sharp thresholds of blow-up and global existence for the coupled nonlinear Schrödinger system*, J. Math. Phys. **49** (2008), no. 6, 062103, 17 pp.
9. M. Nakamura and T. Ozawa, *Nonlinear Schrödinger equations in the Sobolev Space of Critical Order*, J. Funct. Anal. **155**, (1998), no. 2, 364–380.
10. N. V. Nguyen, R. Tian, B. Deconinck, and N. Sheils, *Global existence for a coupled system of Schrödinger equations with power-type non-linearities*, J. Math. Phys. **54** (2013), no. 1, 011503, 19 pp.
11. L. E. Payne and D. H. Sattinger, *Saddle points and instability of nonlinear hyperbolic equations*, Israel J. Math. **22** (1975), no. 3-4, 273–303.
12. B. Ruf, *A sharp Trudinger–Moser type inequality for unbounded domains in \mathbb{R}^2* , J. Funct. Anal. **219**, (2005), no. 2, 340–367.
13. T. Saanouni, *A note on coupled nonlinear Schrödinger equations*, Adv. Nonlinear Anal. **3** (2014), no. 4, 247–269.
14. T. Saanouni, *Global well-posedness and instability of a 2D Schrödinger equation with harmonic potential in the conformal space*, Abstr. Differ. Equ. Appl. **4** (2013), no. 1, 23–42.
15. T. Saanouni, *Remarks on the semilinear Schrödinger equation*, J. Math. Anal. Appl. **400**, (2013), no. 2, 331–344.
16. T. Saanouni, *Decay of solutions to a 2D Schrödinger equation with exponential growth*, J. Partial Differ. Equ. **24**, (2011), no. 1, 37–54.
17. T. Saanouni, *Global well-posedness and scattering of a 2D Schrödinger equation with exponential growth*, Bull. Belg. Math. Soc. Simon Stevin. **17**, (2010), no. 3, 441–462.
18. T. Saanouni, *Scattering for a 2d Schrödinger equation with exponential growth in the conformal space*, Math. Method. App. Sci. **33**, (2010), no. 8, 1046–1058.
19. X. Song, *Stability and instability of standing waves to a system of Schrödinger equations with combined power-type nonlinearities*, J. Math. Anal. Appl. **366**, (2010), no. 1, 345–359.
20. Y. Xu, *Global well-posedness, scattering, and blowup for nonlinear coupled Schrödinger equations in \mathbb{R}^3* , Appl. Anal. **95** (2016), no. 3, 483–502.

UNIVERSITY OF TUNIS EL MANAR, FACULTY OF SCIENCES OF TUNIS, LR03ES04 PARTIAL DIFFERENTIAL EQUATIONS AND APPLICATIONS, 2092 TUNIS, TUNISIA.

E-mail address: hezzi_82hanen@yahoo.fr