

WUR MODULUS AND NORMAL STRUCTURE IN BANACH SPACES

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ABSTRACT. Let X be a Banach space. In this paper, we study the properties of wUR modulus of X , $\delta_X(\varepsilon, f)$, where $0 \leq \varepsilon \leq 2$ and $f \in S(X^*)$, and the relationship between the values of wUR modulus and reflexivity, uniform non-squareness and normal structure, respectively. Among other results, we proved that if $\delta_X(1, f) > 0$, for any $f \in S(X^*)$, then X has weak normal structure.

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