

THE MATRIX POWER MEANS AND INTERPOLATIONS

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ABSTRACT. It is well-known that the Heron mean is a linear interpolation between the arithmetic and the geometric means while the matrix power mean $P_t(A, B) := A^{1/2} \left(\frac{I + (A^{-1/2} B A^{-1/2})^t}{2} \right)^{1/t} A^{1/2}$ interpolates between the harmonic, the geometric, and the arithmetic means. In this article, we establish several comparisons between the matrix power mean, the Heron mean, and the Heinz mean. Therefore, we have a deeper understanding about the distribution of these matrix means.

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