

THE EXISTENCE OF HYPERINVARIANT SUBSPACES FOR WEIGHTED SHIFT OPERATORS

HOSSEIN SADEGHI * and FARZOLLAH MIRZAPOUR

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ABSTRACT. We introduce some classes of Banach spaces for which the hyperinvariant subspace problem for the shift operator has positive answer. Moreover, we provide sufficient conditions on weights which ensure that certain subspaces of $\ell_p^2(\mathbb{Z})$ are closed under convolution. Finally we consider some cases of weighted spaces for which the problem remains open.

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*Corresponding author.

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DEPARTMENT OF MATHEMATICS, UNIVERSITY OF ZANJAN, P.O.BOX 45195-313, ZANJAN,
IRAN.

E-mail address: hsadeghi@znu.ac.ir

E-mail address: f.mirza@znu.ac.ir