

SEMICIRCULAR-LIKE AND SEMICIRCULAR LAWS ON
BANACH *-PROBABILITY SPACES INDUCED BY
DYNAMICAL SYSTEMS OF THE FINITE ADELE RING $A_{\mathbb{Q}}$

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ABSTRACT. Starting from the finite adèle ring $A_{\mathbb{Q}}$, we construct semigroup dynamical systems of $A_{\mathbb{Q}}$, acting on certain C^* -probability spaces. From such dynamical-systematic C^* -probability spaces, we construct Banach-space operators acting on the C^* -probability spaces and corresponding Banach *-probability spaces. In particular, we are interested in Banach-space operators whose free distributions are the (weighted-)semicircular law(s).

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