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# BANACH PARTIAL $*$-ALGEBRAS: AN OVERVIEW

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Abstract. A Banach partial $*$-algebra is a locally convex partial $*$-algebra whose total space is a Banach space. A Banach partial $*$-algebra is said to be of type (B) if it possesses a generating family of multiplier spaces that are also Banach spaces. We describe the basic properties of these objects and display a number of examples, namely, $L^p$-like function spaces and spaces of operators on Hilbert scales or lattices. Finally we analyze the important cases of Banach quasi $*$-algebras and $CQ^*$-algebras.

## References

1. J.-P. Antoine, F. Bagarello, and C. Trapani, *Topological partial $*$-algebras: Basic properties and examples*, Rev. Math. Phys. **11** (1999), 267–302.
2. J.-P. Antoine, A. Inoue, and C. Trapani, *Partial $*$-algebras and their operator rRealizations*, Kluwer, Dordrecht, 2002.
3. J.-P. Antoine and W. Karwowski, *Interpolation theory and refinement of nested Hilbert spaces*, J. Math. Phys. **22** (1981), 2489–2496.
4. J.-P. Antoine and C. Trapani, *Partial inner product spaces – Theory and applications*, Lecture Notes in Mathematics, vol. 1986, Springer-Verlag, Berlin, Heidelberg, 2009.
5. J.-P. Antoine and C. Trapani, *A note on Banach partial $*$-algebras*, Mediterr. J. Math. **3** (2006), 67–86.
6. J.-P. Antoine, C. Trapani, and F. Tschinke, *Continuous $*$-homomorphisms of Banach partial $*$-algebras*, Mediterr. J. Math. **4** (2007), 357–373.
7. F. Bagarello, A. Inoue, and C. Trapani, *Some classes of topological quasi $*$-algebras*, Proc. Amer. Math. Soc. **129** (2001), 2973–2980.


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8. F. Bagarello and C. Trapani, *$CQ^*$-algebras: Structure properties*, Publ. RIMS, Kyoto Univ. **32** (1996), 85–116.
9. F. Bagarello and C. Trapani, *$L^p$-spaces as quasi $*$-algebras*, J. Math. Anal. Appl. **197** (1996,) 810–824.
10. F. Bagarello and C. Trapani, *States and representations of $CQ^*$-algebras*, Ann. Inst. H. Poincaré **61** (1994), 103–133.
11. J. Bergh and J. Löfström, *Interpolation spaces*, Springer-Verlag, Berlin, 1976.
12. G. Birkhoff, *Lattice theory*, 3rd ed., Amer. Math. Soc., Coll. Publ., Providence, RI., 1966.
13. J. J. F. Fournier and J. Stewart, *Amalgams of $L^p$ and $\ell^q$*, Bull. Amer. Math. Soc. **13** (1985), 1–21.
14. G. G. Gould, *On a class of integration spaces*, J. London Math. Soc. **34** (1959), 161–172.
15. R. Haag and D. Kastler, *An algebraic approach to quantum field theorem*, J. Math. Phys. **5** (1964), 848–861.
16. T. Kato, *Perturbation theory for linear operators*, Springer-Verlag, Berlin, 1976.
17. G. Köthe, *Topological Vector Spaces, I and II*, Springer-Verlag, Berlin, 1966, 1979.
18. G. Lassner, *Algebras of unbounded operators and quantum dynamics*, Physica A **124** (1984), 471–480.
19. G. Lassner, *Topological algebras and their applications in quantum statistics*, Wiss. Z. KMU-Leipzig, Math.-Naturwiss. R. **30** (1981), 572–595.
20. E. Nelson, *Note on non-commutative integration*, J. Funct. Anal. **15** (1974), 103–116
21. H. H. Schaefer, *Topological vector spaces*, Springer-Verlag, Berlin, 1971.
22. I. E. Segal, *A noncommutative extension of abstract integration*, Ann. Math. **57** (1953), 401–457.
23. S. Strătilă and L. Zsidó, *Lectures on von Neumann algebras*, Editura Academiei, Bucharest and Abacus Press, Tunbridge Wells, Kent, 1979.
24. C. Trapani, *Bounded elements and spectrum in Banach quasi $*$-algebras*, Studia Math. **172** (2006), 249–273.
25. C. Trapani, *Quasi $*$-algebras of operators and their applications*, Reviews Math. Phys. **7** (1995), 1303–1332.
26. C. Trapani, *States and derivations on quasi $*$-algebras*, J. Math. Phys. **29** (1988), 1885–1890.
27. C. Trapani and M. Fragoulopoulou, *Locally convex quasi $*$-algebras and their representations*, 2018 (in preparation).
28. A. C. Zaanen, *Integration*, North-Holland, Amsterdam, 1967.

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