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CONVOLUTION DOMINATED OPERATORS ON COMPACT EXTENSIONS OF ABELIAN GROUPS

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Abstract. If G is a locally compact group, $CD(G)$ the algebra of convolution dominated operators on $L^2(G)$, then an important question is: Is $\mathbb{C}1 + CD(G)$ (or $CD(G)$ if G is discrete) inverse-closed in the algebra of bounded operators on $L^2(G)$?

In this note we answer this question in the affirmative, provided G is such that one of the following properties is satisfied.

(1) There is a discrete, rigidly symmetric, and amenable subgroup $H \subset G$ and a (measurable) relatively compact neighbourhood of the identity U, invariant under conjugation by elements of H, such that $\{hU \ : \ h \in H\}$ is a partition of G.
(2) The commutator subgroup of G is relatively compact. (If G is connected, this just means that G is an IN group.)

All known examples where $CD(G)$ is inverse-closed in $B(L^2(G))$ are covered by this.

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