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THE STRUCTURE OF FRACTIONAL SPACES GENERATED BY A TWO-DIMENSIONAL NEUTRON TRANSPORT OPERATOR AND ITS APPLICATIONS

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Abstract. In this study, the structure of fractional spaces generated by the two-dimensional neutron transport operator A defined by formula $Au = \omega_1 \frac{\partial u}{\partial x} + \omega_2 \frac{\partial u}{\partial y}$ is investigated. The positivity of A in $C\left(\mathbb{R}^2\right)$ and $L_p\left(\mathbb{R}^2\right)$, $1 \leq p < \infty$, is established. It is established that, for any $0 < \alpha < 1$ and $1 \leq p < \infty$, the norms of spaces $E_{\alpha,p}\left(L_p\left(\mathbb{R}^2\right),\ A\right)$ and $E_\alpha\left(C\left(\mathbb{R}^2\right),\ A\right)$, $W_p^\alpha\left(\mathbb{R}^2\right)$ and $C^\alpha\left(\mathbb{R}^2\right)$ are equivalent, respectively. The positivity of the neutron transport operator in Hölder space $C^\alpha\left(\mathbb{R}^2\right)$ and Slobodeckij space $W_p^\alpha\left(\mathbb{R}^2\right)$ is proved. In applications, theorems on the stability of Cauchy problem for the neutron transport equation in Hölder and Slobodeckij spaces are provided.

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