

## ON SOME INEQUALITIES FOR THE APPROXIMATION NUMBERS IN BANACH ALGEBRAS

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**ABSTRACT.** In this paper, we generalize some inequalities for the approximation numbers of an element in a normed (Banach) algebra  $X$  and, as an application, we present inequalities for the quasinorms of some ideals defined by means of the approximation numbers.

In particular, if  $X = L(E)$  - the algebra of linear and bounded operators  $T : E \rightarrow E$ , where  $E$  is a Banach space, we obtain inequalities for certain quasinorms of operators.

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