

ON TENSORS OF FACTORIZABLE QUANTUM CHANNELS WITH THE COMPLETELY DEPOLARIZING CHANNEL

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Communicated by M. Tomforde

ABSTRACT. In this paper, we obtain results for factorizability of quantum channels. Firstly, we prove that if a tensor $T \otimes S_k$ of a quantum channel T on $M_n(\mathbb{C})$ with the completely depolarizing channel S_k is written as a convex combination of automorphisms on the matrix algebra $M_n(\mathbb{C}) \otimes M_k(\mathbb{C})$ with rational coefficients, then the quantum channel T has an exact factorization through some matrix algebra with the normalized trace. Next, we prove that if a quantum channel has an exact factorization through a finite dimensional von Neumann algebra with a convex combination of normal faithful tracial states with rational coefficients, then it also has an exact factorization through some matrix algebra with the normalized trace.

REFERENCES

1. C. Anantharaman-Delaroche, *On ergodic theorems for free group actions on noncommutative spaces*, Probab. Theory Related Fields **135** (2006), no. 4, 520–546.
2. M. D. Choi, *Completely positive linear maps on complex matrices*, Linear Algebra Appl. **10** (1975), no. 3, 285–290.
3. U. Haagerup and M. Musat, *Factorization and dilation problems for completely positive maps on von Neumann algebras*, Comm. Math. Phys. **303** (2011), no. 2, 555–594.
4. U. Haagerup and M. Musat, *An asymptotic property of factorizable completely positive maps and the Connes embedding problem*, Comm. Math. Phys. **338** (2015), no. 2, 721–752.
5. B. Kümmerer, *Markov dilations on the 2×2 matrices*, In Operator algebras and their connections with topology and ergodic theory (Busteni, 1983), Lect. Notes in Math. **1132**, Berlin, Springer, 1985, 312–323.

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Date: Received: Mar. 29, 2018; Accepted: May 24, 2018.

2010 *Mathematics Subject Classification.* Primary 46L07; Secondary 15A60, 47C15, 47L07.

Key words and phrases. Markov map, factorizable quantum channel, completely depolarizing channel.

6. E. Ricard, *A Markov dilation for self-adjoint Schur multipliers*, Proc. Amer. Math. Soc. **136** (2008), no. 12, 4365–4372.
7. J. A. Smolin, F. Verstraete, and A. Winter, *Entanglement of assistance and multipartite state distillation*, Phys. Rev. A (3) **72** (2005), 0523171–10.

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