

NUMERICAL RADIUS INEQUALITIES FOR OPERATOR MATRICES

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ABSTRACT. Several numerical radius inequalities for operator matrices are proved by generalizing earlier inequalities. In particular, the following inequalities are obtained: if n is even,

$$2w(T) \leq \max\{\|A_1\|, \|A_2\|, \dots, \|A_n\|\} + \frac{1}{2} \sum_{k=0}^{n-1} \| |A_{n-k}|^t |A_{k+1}^*|^{1-t} \|,$$

and if n is odd,

$$2w(T) \leq \max\{\|A_1\|, \|A_2\|, \dots, \|A_n\|\} + w\left(\tilde{A}_{\left(\frac{n+1}{2}\right)_t}\right) + \frac{1}{2} \sum_{k=0}^{n-1} \| |A_{n-k}|^t |A_{k+1}^*|^{1-t} \|,$$

for all $t \in [0, 1]$, A_i 's are bounded linear operators on the Hilbert space \mathcal{H} , and T is off diagonal matrix with entries A_1, \dots, A_n .

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