

## GENERAL EXPONENTIAL DICHOTOMIES: FROM FINITE TO INFINITE TIME

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ABSTRACT. We consider exponential dichotomies on finite intervals and show that if the constants in the notion of an exponential dichotomy are chosen appropriately and uniformly on those intervals, then there exists an exponential dichotomy on the whole line. We consider the general case of a nonautonomous dynamics that need not be invertible. Moreover, we consider both cases of discrete and continuous time.

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