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APPROXIMATION BY CHLODOWSKY VARIANT OF SZÁSZ OPERATORS INVOLVING SHEFFER POLYNOMIALS

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Abstract. In this article, we present a Chlodowsky type variation of Szász operators defined by means of the Sheffer type polynomials. We established convergence properties and the order of convergence through a classical approach, the second order modulus of continuity, Peetre's K-functional, and a new type of weighted modulus of continuity. Furthermore, A-statistical approximation of Korokin type for the operators is also shown and the rate of convergence of operators for functions having derivatives of bounded variation is also obtained. Moreover, some numerical and graphical examples are also given to support our results.

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