

**THE POLAR DECOMPOSITION FOR ADJOINTABLE
OPERATORS ON HILBERT C^* -MODULES
AND CENTERED OPERATORS**

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ABSTRACT. Let T be an adjointable operator between two Hilbert C^* -modules, and let T^* be the adjoint operator of T . The polar decomposition of T is characterized as $T = U(T^*T)^{\frac{1}{2}}$ and $\mathcal{R}(U^*) = \overline{\mathcal{R}(T^*)}$, where U is a partial isometry, $\mathcal{R}(U^*)$ and $\overline{\mathcal{R}(T^*)}$ denote the range of U^* and the norm closure of the range of T^* , respectively. Based on this new characterization of the polar decomposition, an application to the study of centered operators is carried out.

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