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OPERATORS OF LAPLACE TRANSFORM TYPE AND A NEW CLASS OF HYPERGEOMETRIC COEFFICIENTS

STUART BOND and ALI TAHERI*

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Abstract. A differential identity on the hypergeometric function ${}_2F_1(a,b;c;z)$ unifying and extending certain spectral results on the scale of Gegenbauer and Jacobi polynomials and leading to a new class of hypergeometric related scalars $\mathsf{c}_j^m(a,b,c)$ and polynomials $\mathscr{R}_m = \mathscr{R}_m(X)$ is established. The Laplace–Beltrami operator on a compact rank one symmetric space is considered next, and for operators of the Laplace transform type by invoking an operator trace relation, the Maclaurin spectral coefficients of their Schwartz kernel are fully described. Other related representations as well as extensions of the differential identity to the generalized hypergeometric function ${}_pF_q(\mathbf{a};\mathbf{b};z)$ are formulated and proved.

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*Corresponding author .
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Department of Mathematics, University of Sussex, Brighton, East Sussex, UK.
E-mail address: s.bond@sussex.ac.uk
E-mail address: a.taheri@sussex.ac.uk