

DOMINATED ORTHOGONALLY ADDITIVE OPERATORS IN LATTICE-NORMED SPACES

NARIMAN ABASOV¹ and MARAT PLIEV^{2*}

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ABSTRACT. In this paper, we introduce a new class of operators in lattice-normed spaces. We say that an orthogonally additive operator T from a lattice-normed space (V, E) to a lattice-normed space (W, F) is dominated, if there exists a positive orthogonally additive operator S from E to F such that $|Tx| \leq S|x|$ for any element x of (V, E) . We show that under some mild conditions, a dominated orthogonally additive operator has an exact dominant and obtain formulas for calculating the exact dominant of a dominated orthogonally additive operator. In the last part of the paper we consider laterally-to-order continuous operators. We prove that a dominated orthogonally additive operator is laterally-to-order continuous if and only if the same is its exact dominant.

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*Corresponding author .

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¹DEPARTMENT OF MATHEMATICS, MAI – MOSCOW AVIATION INSTITUTE (NATIONAL RESEARCH UNIVERSITY), MOSCOW, 121552 RUSSIA.

E-mail address: abasovn@mail.ru

²LABORATORY OF FUNCTIONAL ANALYSIS, SOUTHERN MATHEMATICAL INSTITUTE OF THE RUSSIAN ACADEMY OF SCIENCES, VLADIKAVKAZ 362027, RUSSIA.

E-mail address: plimarat@yandex.ru