

SOME APPROXIMATION PROPERTIES AND NUCLEAR OPERATORS IN SPACES OF ANALYTICAL FUNCTIONS

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ABSTRACT. We introduce and investigate a new notion of the approximation property $AP_{[c]}$, where $c = (c_n)$ is an arbitrary positive real sequence, tending to infinity. Also, we study the corresponding notion of $[c]$ -nuclear operators in Banach spaces. Some characterization of the $AP_{[c]}$ in terms of tensor products, as well as sufficient conditions for a Banach space to have the $AP_{[c]}$, are given. We give also sufficient conditions for a positive answer to the question: When it follows from the $[c]$ -nuclearity of an adjoint operator the nuclearity of the operator itself. Obtained results are applied then to the study of properties of nuclear operators in some spaces of analytical functions. Many examples are given.

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