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PARTIAL ISOMETRIES AND A GENERAL SPECTRAL THEOREM

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Abstract. We prove a general spectral theorem for an arbitrary densely defined closed linear operator T between complex Hilbert spaces H and K. The corresponding operator measure is partial isometry valued and has properties similar to those of the resolution of the identity of a non-negative self-adjoint operator. The main method is the use of the canonical factorization (polar decomposition) obtained by v. Neumann and Murray. The uniqueness of the generalized resolution of the identity is studied together with the properties of a (non-multiplicative) functional calculus. The properties of this generalized resolution of the identity are also investigated.

References

1. M. S. Birman and M. Z. Solomyak, *Spectral theory of self-adjoint operators in Hilbert space*, Reidel, Dordrecht, 1987.
2. N. Dunford and J. T. Schwartz, *Linear Operators*, Part I, Interscience, New York, 1958; Part II, Interscience, New York, 1963; Part III, Interscience, New York, 1971.
3. I. Erdelyi, *Partial isometries closed under multiplication on Hilbert spaces*, J. Math. Anal. Appl. **22** (1968), 546–551.
4. P. R. Halmos and J. E. McLaughlin, *Partial isometries*, Pacific J. Math. **13** (1963), 585–596.
5. P. R. Halmos and L. J. Wallen, *Powers of partial isometries*, J. Math. Mech. **19** (1970), 657–663.
6. D. Han, D. R. Larson, B. Liu and R. Liu, *Operator-valued measures, dilations, and the theory of frames*, Mem. Amer. Math. Soc. **229** (2014), no. 1075, pp. viii+84.

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7. P. Hines and S. L. Braunstein, *The structure of partial isometries*, in: Semantic techniques in quantum computation, pp. 361–388, Cambridge Univ. Press, Cambridge, 2010.
8. T. Kato, *Perturbation theory for linear operators*, Die Grundlehren der mathematischen Wissenschaften, Band 132 Springer-Verlag New York, Inc., New York, 1966.
9. F. J. Murray, *Linear transformations between Hilbert spaces and the application of this theory to linear partial differential equations*, Trans. Amer. Math. Soc. **37** (1935), 301–338.
10. F. J. Murray, *An introduction to linear transformations in Hilbert space*, Annals of Math. Studies No. 4, Princeton Univ. Press, Princeton, 1941.
11. J. von Neumann, *Über adjungierte Funktionaloperatoren*, Ann. of Math. (2) **33** (1932), no. 2, 294–310.
12. B. Sz.-Nagy, *Spektraldarstellung linearer Transformationen des Hilbertschen Raumes*, (German) Berichtigter Nachdruck. Ergebnisse der Mathematik und ihrer Grenzgebiete, Band 39 Springer-Verlag, Berlin-New York, 1967.
13. C.-Y. Suen, *Completely bounded linear extensions of operator-valued functions on *-semigroups*, Proc. Amer. Math. Soc. **105** (1989), 330–334.
14. J. Weidmann, *Linear operators in Hilbert spaces*, Springer, New York, 1980.

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