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ℓ_1-SUMMABILITY AND LEBESGUE POINTS OF d-DIMENSIONAL FOURIER TRANSFORMS

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Abstract. The classical Lebesgue's theorem is generalized, and it is proved that under some conditions on the summability function θ, the ℓ_1-θ-means of a function f from the Wiener amalgam space $W(L_1, \ell_\infty)(\mathbb{R}^d) \supset L_1(\mathbb{R}^d)$ converge to f at each modified strong Lebesgue point and thus almost everywhere. The θ-summability contains the Weierstrass, Abel, Picard, Bessel, Fejér, de La Vallée-Poussin, Rogosinski, and Riesz summations.

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