

COMPACT EMBEDDINGS ON A SUBSPACE OF WEIGHTED VARIABLE EXPONENT SOBOLEV SPACES

CIHAN UNAL* and ISMAIL AYDIN

Communicated by E. A. Sanchez Perez

ABSTRACT. In this paper, we define an intersection space between weighted classical Lebesgue spaces and weighted Sobolev spaces with variable exponent. We consider the basic properties of the space. Also, we investigate some inclusions, continuous embeddings, and compact embeddings under some conditions.

REFERENCES

1. E. Acerbi and G. Mingione, *Regularity results for a class of functionals with non-standard growth*, Arch. Ration. Mech. and Anal. **156** (2001), 121–140.
2. R. A. Adams and J. J. F. Fournier, *Sobolev spaces (2nd Ed.)*, Academic Press, New York, 2003.
3. I. Aydın, *On variable exponent Amalgam spaces*, An. Ştiinţ. Univ. "Ovidius" Constanţa Ser. Mat. **20** (2012), no. 3, 5–20.
4. I. Aydın, *Weighted variable Sobolev spaces and capacity*, J. Funct. Spaces Appl. **2012** (2012).
5. H. Cartan, *Differential calculus*, Hermann, Paris-France, 1971.
6. P. G. Ciarlet, *Linear and nonlinear functional analysis with applications*, Society for Industrial and Applied Mathematics (SIAM), Philadelphia, 2013.
7. D. V. Cruz-Uribe and A. Fiorenza, *Variable Lebesgue spaces-foundations and harmonic analysis*, Birkhäuser/Springer, New York, 2013.
8. L. Diening, *Maximal function on generalized Lebesgue spaces $L^{p(\cdot)}$* , Math. Inequal. Appl. **7** (2004), no. 2, 245–253.

Copyright 2019 by the Tusi Mathematical Research Group.

Date: Received: Mar. 21, 2018; Accepted: Sep. 18, 2018.

*Corresponding author .

2010 *Mathematics Subject Classification*. Primary 46E35; Secondary 43A15, 46E30.

Key words and phrases. Compact embedding, weighted variable exponent Sobolev space, weighted Lebesgue space.

9. L. Diening, *Riesz potential and Sobolev embeddings on generalized Lebesgue and Sobolev spaces $L^{p(\cdot)}$ and $W^{k,p(\cdot)}$* , Math. Nachr. **268** (2004), 31–43.
10. L. Diening, P. Harjulehto, P. Hästö, and M. Růžička, *Lebesgue and Sobolev spaces with variable exponents*, Springer-Verlag, Berlin, 2011.
11. L. Diening and P. Hästö, *Muckenhoupt weights in variable exponent spaces*, (preprint).
12. E. Edmunds, A. Fiorenza, and A. Meskhi, *On a measure of non-compactness for some classical operators*, Acta Math. Sin. **22** (2006), no. 6, 1847–1862.
13. X. Fan and D. Zhao, *On the spaces $L^{p(x)}(\Omega)$ and $W^{k,p(x)}(\Omega)$* , J. Math. Anal. Appl. **263** (2001), no. 2, 424–446.
14. P. Hajlasz and J. Onninen, *On boundedness of maximal functions in Sobolev spaces*, Ann. Acad. Sci. Fenn. Math. **29** (2004), no. 1, 167–176.
15. K. Ho and I. Sim, *Existence and multiplicity of solutions for degenerate $p(x)$ -Laplace equations involving concave-convex type nonlinearities with two parameters*, Taiwanese J. Math. **19** (2015), no. 5, 1469–1493.
16. K. Ho and I. Sim, *On degenerate $p(x)$ -Laplace equations involving critical growth with two parameters*, Nonlinear Anal. **132** (2016), 95–114.
17. O. D. Kellogg, *Foundations of potential theory*, Springer, Berlin, 1929.
18. Y. Kim, L. Wang, and C. Zhang, *Global bifurcation for a class of degenerate elliptic equations with variable exponents*, J. Math. Anal. Appl. **371** (2010), no. 2, 624–637.
19. O. Kováčik and J. Rákosník, *On spaces $L^{p(x)}$ and $W^{k,p(x)}$* , Czechoslovak Math. J. **41** (1991), no. 4, 592–618.
20. S. Lu, Y. Ding, and D. Yan, *Singular integrals and related topics*, World Scientific Publishing Company, 2007.
21. R. A. Mashiyev, S. Oğraş, Z. Yucedag, and M. Avci, *The Nehari manifold approach for Dirichlet problem involving the $p(x)$ -Laplacian equation*, J. Korean Math. Soc. **47** (2010), no. 4, 845–860.
22. B. Muckenhoupt, *Weighted norm inequalities for the Hardy maximal function*, Trans. Amer. Math. Soc. **165** (1972), 207–226.
23. P. Pucci and Q. Zhang, *Existence of entire solutions for a class of variable exponent elliptic equations*, J. Differ. Equations **257** (2014), no. 5, 1529–1566.
24. M. Růžička, *Electrorheological fluids: Modelling and mathematical theory*, Springer, Berlin, 2000.
25. S. Saiedinezhad and M. B. Ghaemi, *The fibering map approach to a quasilinear degenerate $p(x)$ -Laplacian equation*, Bull. Iran. Math. Soc. **41** (2015), no. 6, 1477–1492.
26. S. Samko, *On a progress in the theory of Lebesgue spaces with variable exponent: maximal and singular operators*, Integral Transforms Spec. Funct. **16** (2005), no. (5-6), 461–482.
27. V. V. Zhikov, *Averaging of functionals of the calculus of variations and elasticity theory*, Math. USSR, Izv. **29** (1987), no. 1, 33–66.

DEPARTMENT OF MATHEMATICS, SINOP UNIVERSITY, 57000, SINOP, TURKEY.

E-mail address: cihanunal88@gmail.com

E-mail address: iaydin@sinop.edu.tr