

TOPOLOGICAL PROPERTIES OF OPERATIONS ON SPACES OF DIFFERENTIABLE FUNCTIONS

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ABSTRACT. In this paper, we consider different notions of openness for the scalar multiplication on sequence spaces and spaces of continuous functions. We apply existing techniques to derive weak openness of multiplication on spaces of differentiable functions, endowed with a large collection of quasi-algebra norms.

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