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M-OPERATORS ON PARTIALLY ORDERED BANACH SPACES

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ABSTRACT. For a matrix $A \in \mathbb{R}^{n \times n}$ whose off-diagonal entries are nonpositive, there are several well-known properties that are equivalent to A being an invertible M-matrix. One of them is the positive stability of A. A generalization of this characterization to partially ordered Banach spaces is considered in this article. Relationships with certain other equivalent conditions are derived. An important result on singular irreducible M-matrices is generalized using the concept of M-operators and irreducibility. Certain other invertibility conditions of M-operators are also investigated.

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