

**MULTIPLICITY OF SOLUTIONS FOR A CLASS
OF NEUMANN ELLIPTIC SYSTEMS IN ANISOTROPIC
SOBOLEV SPACES WITH VARIABLE EXPONENT**

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Communicated by J. D. Rossi

ABSTRACT. In this paper, we prove the existence of infinitely many solutions of a system of boundary value problems involving flux boundary conditions in anisotropic variable exponent Sobolev spaces, by applying a critical point variational principle obtained by Ricceri as a consequence of a more general variational principle and the theory of the anisotropic variable exponent Sobolev spaces.

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Date: Received: Aug. 24, 2018; Accepted: Nov. 2, 2018.

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2010 *Mathematics Subject Classification.* Primary 34A34; Secondary 35D30, 35J50.

Key words and phrases. Neumann elliptic problem, gradient system, weak solution, variational principle, anisotropic variable exponent Sobolev space.

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