

EXISTENCE OF WEAK SOLUTIONS FOR AN INFINITE SYSTEM OF SECOND ORDER DIFFERENTIAL EQUATIONS

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ABSTRACT. In this paper, we investigate the existence of weak solutions for a boundary value problem of an infinite system of second order differential equations. As the main tool, a new Krasnosel'skii type fixed point theorem in Fréchet spaces is established in conjunction with the technique of measures of weak noncompactness.

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