

## EIGENVALUE PROBLEMS INVOLVING THE FRACTIONAL $p(x)$ -LAPLACIAN OPERATOR

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**ABSTRACT.** In this paper, we study a nonlocal eigenvalue problem involving variable exponent growth conditions, on a bounded domain  $\Omega \subset \mathbb{R}^n$ . Using adequate variational techniques, mainly based on Ekeland’s variational principle, we establish the existence of a continuous family of eigenvalues lying in a neighborhood at the right of the origin.

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