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A UNIVERSAL BANACH SPACE WITH A K-UNCONDITIONAL BASIS

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Abstract. For a constant $K \geq 1$, let $\mathfrak{B}_K$ be the class of pairs $(X, (\mathbf{e}_n)_{n\in\omega})$ consisting of a Banach space X and an unconditional Schauder basis $(\mathbf{e}_n)_{n\in\omega}$ for X, having the unconditional basic constant $K_u \leq K$. Such pairs are called K-based Banach spaces. A based Banach space X is rational if the unit ball of any finite-dimensional subspace spanned by finitely many basic vectors is a polyhedron whose vertices have rational coordinates in the Schauder basis of X.

Using the technique of Fraïssé theory, we construct a rational K-based Banach space $\big(\mathbb{U}_K, (\mathbf{e}_n)_{n\in\omega}\big)$ which is $\mathfrak{R}\mathfrak{I}_K$-universal in the sense that each basis preserving isometry $f : \Lambda \to \mathbb{U}_K$ defined on a based subspace Λ of a finite-dimensional rational K-based Banach space A extends to a basis preserving isometry $\bar{f} : A \to \mathbb{U}_K$ of the based Banach space A. We also prove that the K-based Banach space $\mathbb{U}_K$ is almost $\mathfrak{F}\mathfrak{I}_1$-universal in the sense that any base preserving ε-isometry $f : \Lambda \to \mathbb{U}_K$ defined on a based subspace Λ of a finite-dimensional 1-based Banach space A extends to a base preserving ε-isometry $\bar{f} : A \to \mathbb{U}_K$ of the based Banach space A. On the other hand, we show that no almost $\mathfrak{F}\mathfrak{I}_K$-universal based Banach space exists for $K > 1$.

The Banach space $\mathbb{U}_K$ is isomorphic to the complementably universal Banach space for the class of Banach spaces with an unconditional Schauder basis, constructed by Pełczyński in 1969.

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