CHARACTERIZATION OF K-FRAME VECTORS AND K-FRAME GENERATOR MULTIPLIERS

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ABSTRACT. Let $\mathcal{U}$ be a unitary system and let $\mathcal{B}(\mathcal{U})$ be the Bessel vector space for $\mathcal{U}$. In this article, we give a characterization of Bessel vector spaces and local commutant spaces at different complete frame vectors. The relation between local commutant spaces at different complete frame vectors is investigated. Moreover, by introducing multiplication and adjoint on the Bessel vector space for a unital semigroup of unitary operators, we give a C^*-algebra structure to $\mathcal{B}(\mathcal{U})$. Then, we construct some subsets of K-frame vectors that have a Banach space or Banach algebra structure. Also, as a consequence, the set of complete frame vectors for different unitary systems contains Banach spaces or Banach algebras. In the end, we give several characterizations of K-frame generator multipliers and Parseval K-frame generator multipliers.

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