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ATOMIC CHARACTERIZATIONS OF HARDY SPACES ASSOCIATED TO SCHRÖDINGER TYPE OPERATORS

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ABSTRACT. In this article, the authors consider the Schrödinger type operator $L := -\mathrm{div}(A\nabla) + V$ on $\mathbb{R}^n$ with $n \geq 3$, where the matrix A is symmetric and satisfies the uniformly elliptic condition and the nonnegative potential V belongs to the reverse Hölder class $RH_q(\mathbb{R}^n)$ with $q \in (n/2, \infty)$. Let $p(\cdot) : \mathbb{R}^n \to (0, 1]$ be a variable exponent function satisfying the globally log-Hölder continuous condition. The authors introduce the variable Hardy space $H_L^{p(\cdot)}(\mathbb{R}^n)$ associated to L and establish its atomic characterization. The atoms here are closer to the atoms of variable Hardy space $H^{p(\cdot)}(\mathbb{R}^n)$ in spirit, which further implies that $H^{p(\cdot)}(\mathbb{R}^n)$ is continuously embedded in $H_L^{p(\cdot)}(\mathbb{R}^n)$.

REFERENCES

1. D. Albrecht, X. T. Duong, and A. McIntosh, *Operator theory and harmonic analysis*, in: *Instructional Workshop on Analysis and Geometry, Part III (Canberra, 1995)*, 77–136, Proc. Centre Math. Appl. Austral. Nat. Univ., **34**, Austral. Nat. Univ., Canberra, 1996.
2. P. Auscher, X. T. Duong, and A. McIntosh, *Boundedness of Banach space valued singular integral operators and Hardy spaces*, unpublished manuscript, 2005.
3. P. Auscher and E. Russ, *Hardy spaces and divergence operators on strongly Lipschitz domains of* $\mathbb{R}^n$, J. Funct. Anal. **201** (2003), no. 1, 148–184.
4. P. Auscher and P. Tchamitchian, *Square root problem for divergence operators and related topics*, Astérisque No. 249 (1998), viii+172 pp.

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5. J. Cao, D.-C. Chang, D. Yang, and S. Yang, *Boundedness of second order Riesz transforms associated to Schrödinger operators on Musielak-Orlicz-Hardy spaces*, Commun. Pure Appl. Anal. **13** (2014), no. 4, 1435–1463.
6. R. R. Coifman, *A real variable characterization of H^p*, Studia Math. **51** (1974), 269–274.
7. R. R. Coifman, Y. Meyer, and E. M. Stein, *Some new function spaces and their applications to harmonic analysis*, J. Funct. Anal. **62** (1985), no. 2, 304–335.
8. D. V. Cruz-Uribe and A. Fiorenza, *Variable Lebesgue Spaces*, Foundations and Harmonic Analysis, Applied and Numerical Harmonic Analysis, Birkhäuser/Springer, Heidelberg, 2013.
9. D. Cruz-Uribe, A. Fiorenza, J. M. Martell, and C. Pérez, *The boundedness of classical operators on variable L^p spaces*, Ann. Acad. Sci. Fenn. Math. **31** (2006), no. 1, 239–264.
10. D. Cruz-Uribe and L.-A. D. Wang, *Variable Hardy spaces*, Indiana Univ. Math. J. **63** (2014), no. 2, 447–493.
11. L. Diening, P. Harjulehto, P. Hästö, and M. Růžička, *Lebesgue and Sobolev Spaces with Variable Exponents*, Lecture Notes in Mathematics, 2017, Springer, Heidelberg, 2011.
12. X. T. Duong and L. Yan, *New function spaces of BMO type, the John-Nirenberg inequality, interpolation, and applications*, Comm. Pure Appl. Math. **58** (2005), no. 10, 1375–1420.
13. X. T. Duong and L. Yan, *Duality of Hardy and BMO spaces associated with operators with heat kernel bounds*, J. Amer. Math. Soc. **18** (2005), no. 4, 943–973.
14. J. Dziubański and J. Zienkiewicz, *Hardy space H^1 associated to Schrödinger operator with potential satisfying reverse Hölder inequality*, Rev. Mat. Iberoamericana **15** (1999), no. 2, 279–296.
15. J. Dziubański and J. Zienkiewicz, *H^p spaces for Schrödinger operators, in Fourier analysis and related topics*, Banach Center Publ., **56**, Polish Acad. Sci. Inst. Math., Warsaw, (2002), 45–53.
16. J. Dziubański and J. Zienkiewicz, *H^p spaces associated with Schrödinger operators with potentials from reverse Hölder classes*, Colloq. Math. **98** (2003), no. 1, 5–38.
17. C. Fefferman and E. M. Stein, *H^p spaces of several variables*, Acta Math. **129**, 1972, no. 3-4, 137–193.
18. S. Hofmann, G. Lu, D. Mitrea, M. Mitrea, and L. Yan, *Hardy Spaces Associated to Non-negative Self-adjoint Operators Satisfying Davies-Gaffney Estimates*, Mem. Amer. Math. Soc. **214** (2011), no. 1007, vi+78 pp.
19. S. Hofmann and S. Mayboroda, *Hardy and BMO spaces associated to divergence form elliptic operators*, Math. Ann. **344** (2009), no. 1, 37–116.
20. R. Jiang and D. Yang, *New Orlicz-Hardy spaces associated with divergence form elliptic operators*, J. Funct. Anal. **258** (2010), no. 4, 1167–1224.
21. T. Kato, *Perturbation Theory for Linear Operators*, Reprint of the 1980 edition, Classics in Mathematics, Springer-Verlag, Berlin, 1995.
22. R. H. Latter, *A decomposition of $H^p(\mathbb{R}^n)$ in terms of atoms*, Studia Math., **62** (1978), no. 1, 93–101.
23. E. Nakai and Y. Sawano, *Hardy spaces with variable exponents and generalized Campanato spaces*, J. Funct. Anal. **262** (2012), no. 9, 3665–3748.
24. Y. Sawano, *Atomic decompositions of Hardy spaces with variable exponents and its application to bounded linear operators*, Integral Equations Operator Theory **77** (2013), no. 1, 123–148.
25. Z. Shen, *L^p estimates for Schrödinger operators with certain potentials*, Ann. Inst. Fourier (Grenoble) **45** (1995), no. 2, 513–546.
26. E. M. Stein and G. Weiss, *On the theory of harmonic functions of several variables. I. The theory of H^p-spaces*, Acta Math. **103** (1960), 25–62.
27. L. Yan, *Classes of Hardy spaces associated with operators, duality theorem and applications*, Trans. Amer. Math. Soc. **360** (2008), no. 8, 4383–4408.
28. S. Yang, *Some estimates for Schrödinger type operators on Musielak-Orlicz-Hardy spaces*, Taiwanese J. Math. **18** (2014), no. 4, 1293-1328.

29. D. Yang and J. Zhang, *Variable Hardy spaces associated with operators satisfying Davies-Gaffney estimates on metric measure spaces of homogeneous type*, Ann. Acad. Sci. Fenn. Math. **43** (2018), no. 1, 47–87.
30. D. Yang, J. Zhang, and C. Zhuo, *Variable Hardy spaces associated with operators satisfying Davies-Gaffney estimates*, Proc. Edinb. Math. Soc. **61** (2018), no. 3, 759–810.
31. C. Zhuo, D. Yang, and Y. Liang, *Intrinsic square function characterizations of Hardy spaces with variable exponents*, Bull. Malays. Math. Sci. Soc. **39** (2016), no. 4, 1541–1577.

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