

## SPECIAL FACTORS OF INVERTIBLE ELEMENTS IN SIMPLE UNITAL PURELY INFINITE $C^*$ -ALGEBRAS

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ABSTRACT. In simple unital purely infinite  $C^*$ -algebra  $A$ , Leen proved that any element in the identity component of the invertible group is a finite product of symmetries of  $A$ . Revising Leen’s factorization, we show that a multiple of eight of such factors are  $*$ -symmetries of the form  $1 - 2P_{i,j}(u)$ , where  $P_{i,j}(u)$  are certain projections of the  $C^*$ -matrix algebra, defined by Dye as

$$P_{i,j}(u) = \frac{1}{2}(e_{i,i} + e_{j,j} + e_{i,1}ue_{1,j} + e_{j,1}u^*e_{1,i}),$$

for a given system of matrix units  $\{e_{i,j}\}_{i,j=1}^n$  of  $A$  and a unitary  $u \in \mathcal{U}(A)$ .

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