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CLASS OF OPERATORS WITH SUPERIORLY CLOSED NUMERICAL RANGES

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Abstract. The aim of this paper is to introduce a class of operators acting on a complex Hilbert space. This class contains, among others, nonzero compact operators. We give a characterization of this class in term of generalized numerical ranges and deduce that if A is a compact operator, then $w(A) = |\lambda|$ with $\lambda \in W(A)$, where $W(A)$ and $w(A)$ are the numerical range and the numerical radius of A, respectively. We will give some new necessary conditions for an operator to be compact. We also show some light on the generalized numerical ranges of the elementary operators $\delta_{2,A,B}$ and $\mathcal{M}_{2,A,B}$.

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