

ON ZIPF–MANDELBROT ENTROPY AND 3-CONVEX FUNCTIONS

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ABSTRACT. In this paper, we present some interesting results related to the bounds of Zipf–Mandelbrot entropy and the 3-convexity of the function. Further, we define linear functionals as the nonnegative differences of the obtained inequalities and we present mean value theorems for the linear functionals. Finally, we discuss the n -exponential convexity and the log-convexity of the functions associated with the linear functionals.

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