

ANALYTIC VARIABLE EXPONENT HARDY SPACES

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ABSTRACT. We introduce a variable exponent version of the Hardy space of analytic functions on the unit disk. We then show some properties of the space and give an example of a variable exponent $p(\cdot)$ that satisfies the log-Hölder condition and $H^{p(\cdot)} \neq H^q$ for every constant exponent $q \in (1, \infty)$. We also consider a variable exponent version of the Hardy space on the upper-half plane.

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