

RIESZ TRANSFORM AND FRACTIONAL INTEGRAL OPERATORS GENERATED BY NONDEGENERATE ELLIPTIC DIFFERENTIAL OPERATORS

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Dedicated to Professor Alan McIntosh

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ABSTRACT. The Morrey boundedness is proved for the Riesz transform and the inverse operator of the nondegenerate elliptic differential operator of divergence form generated by a vector-function in $(L^\infty)^{n^2}$, and for the inverse operator of the Schrödinger operators whose nonnegative potentials satisfy a certain integrability condition. In this note, our result is not obtained directly from the estimates of integral formula, which reflects the fact that the solution of the Kato conjecture did not use any integral expression of the operators. One of the important tools in the proof is the decomposition of the functions in Morrey spaces based on the elliptic differential operators in question. In some special cases where the integral kernel comes into play, the boundedness property of the Littlewood–Paley operator was already obtained by Gong. So, the main novelties of this paper are the decomposition results associated with elliptic differential operators and the result in the case where the explicit formula of the integral kernel of the heat semigroup is unavailable.

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