

A TRICK FOR INVESTIGATION OF NEAR-MARTINGALES IN QUANTUM PROBABILITY SPACES

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Communicated by U. C. Ji

ABSTRACT. We introduce near-martingales in the setting of quantum probability spaces and present a trick for investigating some of their properties. For instance, we give a near-martingale analogous result of the fact that the space of all bounded L^p -martingales, equipped with the norm $\|\cdot\|_p$, is isometric to $L^p(\mathfrak{M})$ for $p > 1$. We also present Doob and Riesz decompositions for the near-submartingale and provide Gundy's decomposition for L^1 -bounded near-martingales. In addition, the interrelation between near-martingales and the instantly independence is studied.

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Date: Received: Feb. 18, 2019; Accepted: Apr. 1, 2019.

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2010 *Mathematics Subject Classification.* Primary 46L53; Secondary 46L10, 60E15, 47A30.

Key words and phrases. Quantum probability space, Gundy decomposition, noncommutative near-martingale, Doob decomposition, Riesz decomposition.

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