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APPROXIMATE AND TRAJECTORY CONTROLLABILITY OF FRACTIONAL NEUTRAL DIFFERENTIAL EQUATION

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ABSTRACT. We study a new class of fractional neutral differential control system with noninstantaneous impulses and state-dependent delay. The resolvent family and Krasnoselskii's fixed point theorem are utilized to examine the approximate controllability outcomes for the proposed system. Further, we derive the trajectory controllability outcomes for the proposed fractional control system. Finally, the main results are validated with the aid of an example.

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