

THREE SOLUTIONS FOR A KIRCHHOFF-TYPE PROBLEM INVOLVING NONLOCAL FRACTIONAL p -LAPLACIAN

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ABSTRACT. Using the three critical points theorem, we obtain the existence of three weak solutions for a Kirchhoff-type problem involving the nonlocal fractional p -Laplacian operator in a fractional Sobolev space, with homogeneous Dirichlet boundary conditions.

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